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SOME REMARKS ON THE NUMERICAL SOLUTION OF TRICOMI-TYPE EQUATIONS

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C. K. Chu[†], L. W. Xiang and Z. K. Yao

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ABSTRACT

Some aspects of the numerical solution of the Tricomi equation

$$y\phi_{xx} - \phi_{yy} = 0 \tag{1}$$

and the inverted Tricomi equation

$$y\phi_{yy} - \phi_{xx} = 0 \tag{2}$$

with particular emphasis on periodic problems are studied.

AMS(MOS) Subject Classification: 35M05, 65P05, 65N05, 65N10, 76H05

Key Words: Tricomi equation, mixed elliptic-hyperbolic equations, periodic problems, numerical methods

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SIGNIFICANCE AND EXPLANATION

We consider in this paper some aspects of the numerical solution of the Tricomi equation

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with particular emphasis on periodic problems. These periodic problems are of actual physical interest: the former is a model for the deflection of a floppy disc considered as a rotating membrane, while the latter is a model for the transonic deLaval nozzle. Since most studies of the Tricomi equations have been in domains bounded by one or more characteristics, such periodic problems offer some different viewpoints and some different qualitative insight into these mixed elliptic-hyperbolic equations.

We shall test out various numerical schemes on these two problems. These equations being linear, many mathematical questions, e.g. unique solvability, convergence, etc., can be easily answered on these model problems that would help shed light on actual nonlinear numerical procedures, say, those used . ..ansonic flow. We shall concentrate on the unique solvability of the algebraic equations, and expect convergence etc. can be studied equally readily.

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the authors of this report.

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1. INTRODUCTION

We consider in this paper some aspects of the numerical solution of the Tricomi equation

$$y\phi_{xx} - \phi_{yy} = 0 \tag{1}$$

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2. THE PERIODIC TRICOMI PROBLEM AND FLOPPY DISC

A rotating circular membrane clamped at the inner edge $r = a$ and loaded with a transverse force (fig. 1) is described by the following equation |1| :

$$- (1-r^2)\, w_{rr} - \frac{1}{r}\,(3r^2- 1)\, w_r + \frac{1}{r^2}\,(3r^2- 1)\, w_{\theta\theta} = F \quad (3)$$

Here w is the (small) deflection transverse to the disc, θ is the usual angular coordinate, r the radial coordinate normalized with respect to the outer radius, and the driving term F contains the loading force per unit area, angular velocity, disc thickness, and material constants. The appropriate boundary conditions are

$$\begin{aligned} w\,(a,\theta) &= 0 \\ w\,(1,\theta) &< \infty \\ w\,(r,\theta) &= w(r,\theta + 2\pi) \end{aligned} \quad (4)$$

expressing, respectively, clamping at the inner edge, finite deflection at the outer edge, and periodicity in θ. The equation is very interesting in that it is elliptic in the region $a < r < 1/\sqrt{3}$, hyperbolic in the region $1/\sqrt{3} < r < 1$, and degenerate parabolic on the transition line $r = 1/\sqrt{3}$ and again on the outer edge $r = 1$. The characteristics in the hyperbolic region have the general shapes shown in fig. 1.

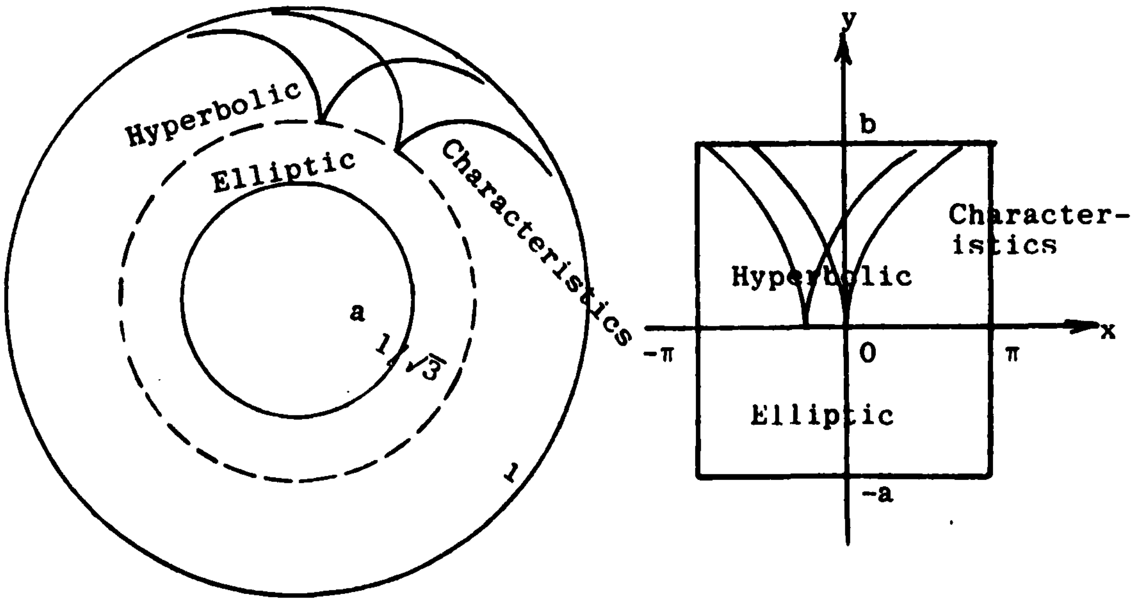

Fig. 1. (Left) Geometry of floppy disc problem
(Right) Model using Tricomi equation

This problem can obviously be modeled by the following periodic problem for the Tricomi equation: ϕ is to satisfy equation (1) in the domain $-\infty < x < +\infty$, $-a < y < +b$ and the boundary conditions

$$\begin{aligned} &\phi \text{ (or } \phi_y) \text{ prescribed at } y = -a, \quad y = +b \\ &\phi(x,y) = \phi(x+2\pi, y) \end{aligned} \tag{5}$$

The modeling is not exact in that the inessential first derivative term in (3) is discarded and that the outer boundary $y = +b$ is not a degenerate parabolic line any more. Other than that, the essential features, including the shape of the characteristics, are all properly modeled. The inhomogeneous differential equation (3) with homogeneous boundary conditions (4) has also been changed to the homogeneous equation (1) and inhomogeneous boundary conditions (5), the equivalence of these problems being well known.

The problem is obviously treated by separation of variables. Letting

$$\phi = \phi_n e^{inx} ,$$

the Fourier component ϕ_n for each wave number n $(n > 0)$ satisfies an Airy equation

$$\phi_n'' + n^2 y \phi_n = 0 \tag{6}$$

The solution to (6) is given in terms of the Airy functions

$$\phi_n = A_n \operatorname{Ai}(z) + B_n \operatorname{Bi}(z) , \tag{7}$$

where $z = n^{-2/3} y$, and A_n, B_n are arbitrary constants. For $n = 0$, $\phi_o = A_o + B_o y$ trivially. For large z, the functions $\operatorname{Ai}(z)$ and $\operatorname{Bi}(z)$ behave like slightly damped exponentials or slightly damped sinusoids, depending on whether z is positive or negative:

$$\begin{aligned} \operatorname{Ai}(z) &\sim \frac{1}{2\sqrt{\pi}} z^{-1/4} e^{-(2/3)z^{3/2}} , \quad z > 0 \\ &\sim -\frac{1}{\sqrt{\pi}}(-z)^{-1/4} \sin\{\tfrac{2}{3}(-z)^{3/2} + \tfrac{\pi}{4}\} , \quad z < 0 \\ \operatorname{Bi}(z) &\sim \frac{1}{\sqrt{\pi}} z^{-1/4} e^{+(2/3)z^{3/2}} , \quad z > 0 \\ &\sim -\frac{1}{\sqrt{\pi}}(-z)^{-1/4} \cos\{\tfrac{2}{3}(-z)^{3/2} + \tfrac{\pi}{4}\} , \quad z < 0 \end{aligned} \tag{8}$$

The boundary conditions to be satisfied by ϕ_n are the n-th Fourier coefficients of the given data at $y = -a$ and $+b$. Thus, the problem is uniquely solvable if and only if the determinant

$$\begin{vmatrix} Ai(n^{2/3}a) & Bi(n^{2/3}a) \\ Ai(n^{-2/3}b) & Bi(n^{-2/3}b) \end{vmatrix} \neq 0 \qquad (9)$$

For large n, this is approximately equivalent to

$$\frac{1}{2} e^{(4/3)na^{3/2}} \cot\left(\frac{2}{3} nb^{3/2} + \frac{\pi}{4}\right) \neq 1 .$$

For given domains, condition (9) will be violated for some very large n. Thus one may conclude that the floppy disc problem admits no unique solution. However, the membrane approximation in equation (3) is really only valid for n not too large, say, $n \leq N$; since if the azimuthal wave lengths become too small compared to the radial dimension of the disc, the membrane approximation fails as bending must now be taken into account. With the limitation of $n \leq N$, then the periodic problem in general has a unique solution provided (9) is satisfied. For example, if $a = b = 1$, actual calculation shows that (9) is still satisfied for n as high as 100.

3. THE INVERTED TRICOMI EQUATION AND THE NOZZLE PROBLEM

The converging-diverging nozzle, in which a gas expands from subsonic velocity to supersonic velocity, can obviously be viewed also as a periodic problem, if we introduce potential or streamline coordinates, in which the walls would be either constant function values or zero normal derivatives and thus the function can be extended to a periodic function in the usual elementary way. If, as is often tempting to do, we try to think of modeling the nozzle flow by the Tricomi equation, then the arguments of the previous section would convince us that two boundary conditions are needed, one at the subsonic upstream and one at the supersonic downstream. But in fact, this is never the case in practice: a supersonic boundary condition is never prescribed (see e.g. |2|). The "physical"argments that signals do not propagate upstream are inadequate to explain this discrepancy.

The real reason is that the Tricomi equation is the wrong model to use for transonic flow in the physical plane, although it is the correct model in the hodograph plane. The characteristics in the supersonic flow region, fig. 2, are perpendicular to the streamlines at the sonic transition, and therefore, they are almost tangent to the transition line. The characteristics of the Tricomi equation, on the contrary, are perpendicular to the transition line, as in fig. 1. Thus for calculating in the physical plane, the Tricomi equation does not model the nozzle flow problem correctly. In fact, the inverted Tricomi equation (2) is the right model to study. In his book |3| , Tricomi also considered the inverted Tricomi equation but stated that the equation did not seem to have a physical application. The present model appears to be indeed such an application.

The proper periodic boundary value problem for the inverted Tricomi equation (2) is: ϕ is to satisfy (2) in the domain $-\infty < x < \infty$, $-a < y < b$ and the boundary conditions

$$\begin{aligned} &\phi \text{ or } \phi_y \text{ prescribed at } y = -a \\ &\text{No data to be prescribed at } y = +b \\ &\phi(x,y) = \phi(x+2\pi, y) \end{aligned} \tag{10}$$

Again letting $\phi = \phi_n e^{inx}$, the equation satisfied by each ϕ_n is now

$$y\phi_n'' + n^2 \phi_n = 0 \tag{11}$$

In the neighborhood of $y = 0$, elementary series methods give two linearly independent solutions for each $n \neq 0$:

$$\begin{aligned} \phi_n^{(1)} &= y\left(1 - \frac{n^2}{2!} y + \frac{n^4}{3!2!} y^2 - \frac{n^6}{4!3!} y^3 + \ldots\right) \\ \phi_n^{(2)} &= \phi_n^{(1)} \log y \end{aligned} \tag{12}$$

Since the derivatives of model velocity components, we require that they be square-integrable. The solution $\phi_n^{(2)}$ fails to satisfy this requirement, and must be rejected. We are then left with one solution in (12), corresponding to the one boundary condition at $y = -a$ and no boundary condition at $y = +b$. That the converse cannot be prescribed is based on the ellipticity or hyperbolicity of the original partial differential equation, and cannot be seen from such a simple argument as just presented.

On the other hand, for $n = 0$, we have $\phi_o = C_1 + C_2 y$. C_2 is determined from the Fourier coefficients of the given data at $y = -a$, just as all the coefficients of the ϕ_n, $n \neq 0$, are determined. C_1 is determined by prescribing the average value of $\phi(x,0)$ on the transition line, where $\phi_{xx} = 0$ and ϕ is a linear function in x.

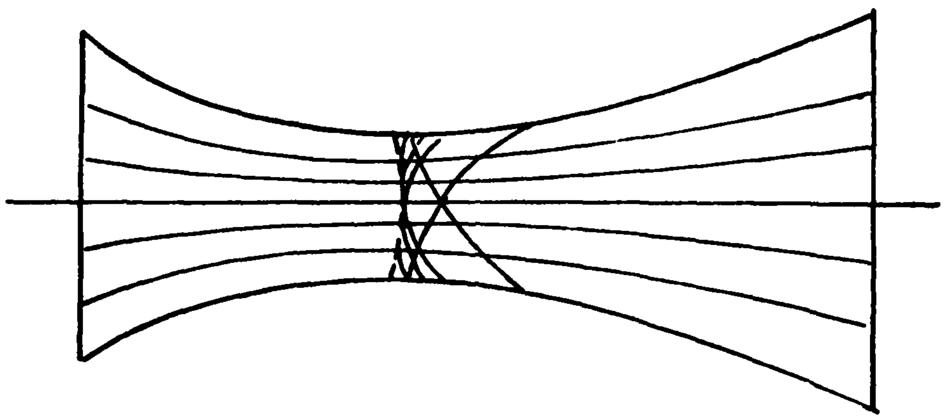

Fig. 2. Sketch of deLaval nozzle showing transition line and characteristics.

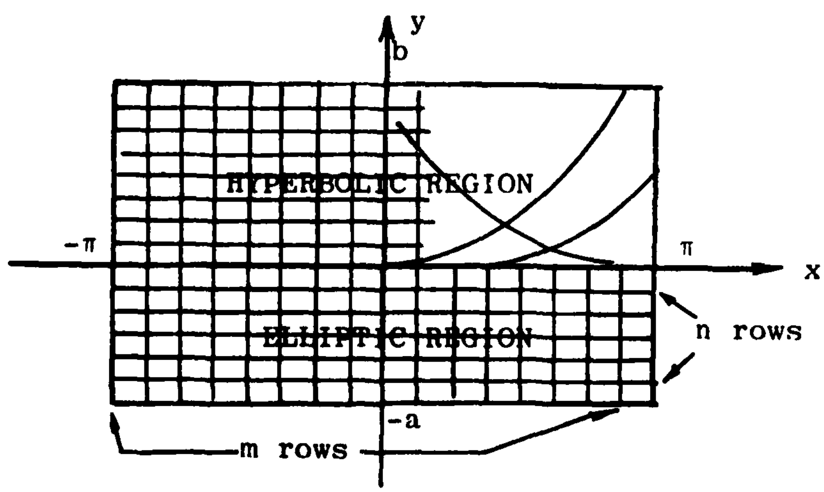

Fig. 3. Domain and grid for inverted Tricomi equation in model nozzle problem.

4. FINITE DIFFERENCE SOLUTION OF THE MODEL NOZZLE PROBLEM

We apply the method of Murman and Cole |4| to the model nozzle problem with the inverted Tricomi equation. We lay down a finite difference grid (fig. 3) of grid width Δx and Δy respectively, in which the line $y = 0$ is a row of grid points, to start; later on this inessential restriction will be removed. The Murman-Cole method uses five-point centered differences if the center point lies in the elliptic region (as in a Laplace equation), and uses backward second differences in y if the point in question lies in the hyperbolic region (as in an implicit scheme for the wave equation). Thus, we have

$$
\begin{aligned}
y\phi_{yy} - \phi_{xx} &\sim \frac{y_j(\phi_{i,j+1} + \phi_{i,j-1} - 2\phi_{i,j})}{\Delta y^2} - \\
&\quad - \frac{\phi_{i+1,j} + \phi_{i-1,j} - 2\phi_{i,j}}{\Delta x^2} \quad \text{for } y_j < 0 \\
&\sim \frac{y_j(\phi_{i,j} + \phi_{i,j-2} - 2\,\phi_{i,j-1})}{\Delta y^2} \\
&\quad - \frac{\phi_{i+1,j} + \phi_{i-1,j} - 2\phi_{i,j}}{\Delta x^2} \quad \text{for } y_j > 0
\end{aligned}
\tag{13}
$$

We shall prescribe boundary conditions at $y = -a$, and prescribe periodic boundary conditions on the side walls. The transition line being a row of grid points, the difference equations for these points degenerate to $\phi_{xx} = 0$, exactly as does the differential equation. This, together with the side-wall periodicity requirements, yields ϕ = const on $y = 0$. From what was said about the $n = 0$ case in the previous section, we are permitted to prescribe this constant to, say, $\phi(x,0) = 0$.*

In this simple case, the elliptic and hyperbolic regions completely uncouple.* The difference equations in the elliptic region possess a unique solution, as we shall show below. The situation is not much different from the Laplace equation,

---

* The model is too restrictive compared to the real nozzle, in this sense. The transition line in the real nozzle is almost a characteristic, whereas in the model it is exactly a characteristic.

since the degeneracy on the x-axis has been properly taken care of. Once the unique solution is obtained in the elliptic region, it is trivial to show that the equations in the hyperbolic region have unique solutions line by line (standard implicit scheme for wave equation).

To prove unique existence for the elliptic region, let the elliptic region have m rows of interior grid points in the x-direction, and n rows in the y-direction, fig. 3. The difference equations for this region become

$$A\underline{\phi} = \underline{f} \tag{14}$$

where the column vector $\underline{\phi}$ denotes $(\phi_{11}, \phi_{12}, \ldots \phi_{1n}, \phi_{21} \ldots \phi_{mn})^T$ and the column vector $\underline{f}$ denotes the boundary data as usual. The matrix A has the form

$$A = \frac{1}{\Delta x^2} \begin{bmatrix} Q & -I & 0 & 0 & \ldots\ldots 0 & -I \\ -I & Q & -I & 0 & \ldots\ldots 0 & 0 \\ 0 & -I & Q & -I & \ldots\ldots 0 & 0 \\ & \ldots\ldots\ldots\ldots & & & & \\ -I & 0 & 0 & 0 & \ldots\ldots -I & Q \end{bmatrix} \tag{15}$$

where submatrices Q and I are $n \times n$ matrices, and there are $m \times m$ of them to make up A.

For Neumann data on $y = -a$, the matrix Q is

$$Q = \begin{bmatrix} nr+2 & -nr & 0 & 0 & \ldots\ldots\ldots & 0 \\ -(n-1)r & 2(n-1)r+2 & -(n-1)r & 0 & \ldots\ldots\ldots & 0 \\ 0 & -(n-2)r & 2(n-2)r+2 & -(n-2)r & \ldots & 0 \\ & \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & & & & \\ 0 & 0 & 0 \ldots\ldots & -2r & 4r+2 & -2r \\ 0 & 0 & 0 \ldots\ldots & 0 & -r & 2r+2 \end{bmatrix} \tag{16}$$

where $r = \Delta x^2 / \Delta y$. For Dirichlet data on $y = -a$, Q is the same matrix as (16), except that the first element Q_{11} is replaced by $2nr+2$.

We now use a theorem of Taussky-Todd |5| , which states that for irreducible matrices M, with positive diagonal elements, $M_{\lambda\lambda} > 0$, and nonpositive off-diagonal elements, $M_{\lambda\mu} \leq 0$

for $\lambda \neq \mu$, M is singular if and only if $\sum_\mu M_{\lambda\mu} = 0$ for every λ . Our matrix A for both Neumann and Dirichlet data on $y = -a$ satisfies all these conditions, except $\sum_\mu A_{\lambda\mu} \neq 0$ for $\lambda = n, 2n, \ldots mn$. Hence A is nonsingular, and the unique solvability of the difference equations in the elliptic region is proved.

The condition that $y = 0$ be a row of grid points will now be removed. To this end, let the transition line $y = 0$ be at a distance $\alpha\Delta y$ from the last elliptic row $(j = n)$ and a distance of $(1-\alpha)\Delta y$ from the first hyperbolic row $(j = n+1)$, where $0 < \alpha < 1$. Defining $(1-\alpha)\phi_{i,n} + \phi_{i,n+1} = \hat{\phi}_i$, substituting $j = n$ and $j = n+1$ into the first and second forms of (13) respectively, and adding them properly, we get

$$\hat{\phi}_{i+1} + \hat{\phi}_{i-1} - 2\hat{\phi}_i = 0 .$$

Obviously, $\hat{\phi}_i$ is the interpolated value for $\phi(x,0)$, and this equation approximates $\phi_{xx} = 0$ on $y = 0$. This equation should be solved first, again with periodicity conditions on the side walls, to give $\hat{\phi}_i = \text{const} = 0$ as in the previous case.

Then we replace $\phi_{i,n+1}$ by $-\frac{1-\alpha}{\alpha}\phi_{i,n}$, and again have just a mn x mn system of linear equations in the elliptic region. Matrix A once more has the form given in (15), except that Q (for Neumann data on $y = -a$) is now

$$Q = \begin{bmatrix} (n-1+\alpha)r+2 & -(n-1+\alpha)r & 0 & 0 \ldots. & 0 \\ -(n-2+\alpha)r & 2(n-2+\alpha)r+2 & -(n-2+\alpha)r & 0 \ldots. & 0 \\ & \multicolumn{3}{c}{\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots} & \\ 0 & 0 \ldots\ldots & -(1+\alpha)r & 2(1+\alpha)r+2 & -(1+\alpha)r \\ 0 & 0 \ldots\ldots & 0 & -\alpha r & (1+\alpha)r+2 \end{bmatrix}$$

Again, the theorem of Taussky-Todd insures the nonsingularity of A. A similar argument also holds for Dirichlet data on $y = -a$.

We conclude this section with some computed results. Fig. 4 shows a model nozzle for the inverted Tricomi equation, calculated with $\phi = 0$ on the transition line, periodic boundary conditions on the side walls, and two cases of Neumann data on $y = -a$: (i) $\phi_y = 1 + \cos x$, and (ii) $\phi_y = 1 + \cos x + \sin x$. Δx and Δy were taken to be $\pi/8$ and 1/5 respectively, giving $n = 4$ and $m = 8$ in the elliptic region. Presented in the figure are values of ϕ_y (modeling the axial velocity) as functions of x at various values of y. The solid curves represent the analytic solution described in Sec. 3, while the circles are the numerical solution. The accuracy is obviously very satisfactory.

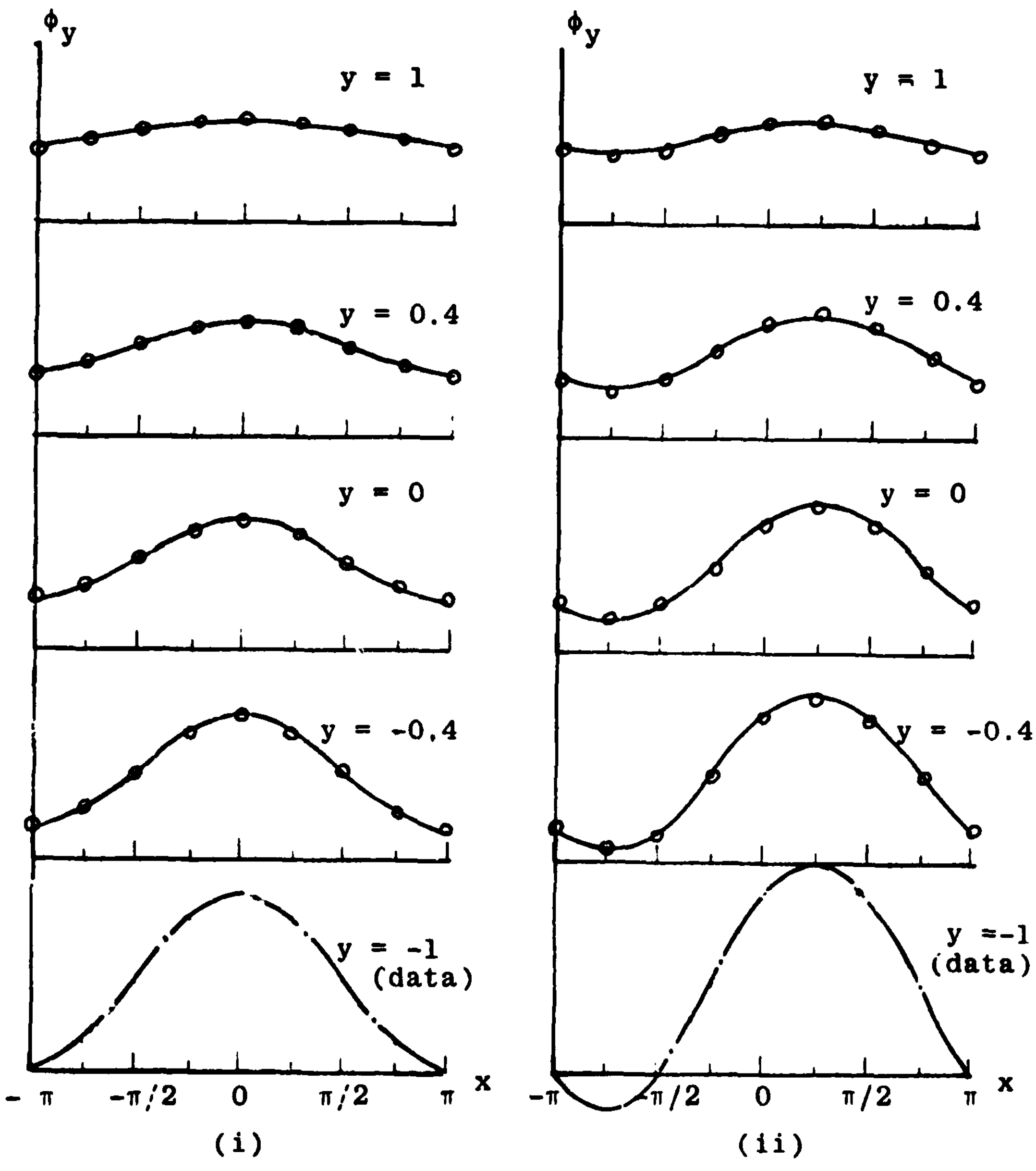

Fig. 4. Calculated model nozzle flow compared with exact solution.

3. FINITE DIFFERENCE SOLUTION FOR THE PERIODIC TRICOMI PROBLEM (FLOPPY DISC MODEL)

For the periodic problem of the Tricomi equation, as described in Section 2, we recall that boundary conditions are needed at both $y = -a$ and $y = +b$, and that a unique solution exists only when the domain does not admit eigensolutions corresponding to wave numbers $n \leq N$. An obvious method to solve such problems is indeed to solve equation (6) as an ordinary differential equation, and treat the eigenfunctions as part of that problem. Here we propose, however, an alternate procedure, starting directly with a finite difference grid for the partial differential equation.

We denote by $\phi_0(x)$ and $\phi_2(x)$ the boundary values given on $y = -a$ and $y = +b$ respectively, and we denote by $\phi_1(x)$ the function $\phi(x,0)$ on the transition line. If ϕ_1 were known, the solution of the difference equations in the elliptic region will exist uniquely, since the matrix A will be nonsingular for exactly the same reasons as with the inverted Tricomi equation discussed in the previous section. Then we can apply the Murman-Cole scheme in the hyperbolic region as before, and obtain the entire solution up to $\phi(x,b)$, which should be equal to $\phi_2(x)$. But $\phi(x,b)$ is a linear transformation of $\phi_0(x)$ and of $\phi_1(x)$ summed, which gives precisely a condition to determine $\phi_1(x)$:

$$\phi(x,b) = S\phi_0 + T\phi_1 = \phi_2$$

Or,

$$\phi_1 = T^{-1}(\phi_2 - S\phi_0) \tag{17}$$

Here S and T are the matrices representing the linear transformations, and the method works only if T has an inverse. This condition is exactly the finite difference counterpart of the nonoccurrence of the eigenfunctions.

In actual computations, we first solve the entire problem with $\phi_1(x) = 0$, and get $\phi(x,b) = S\phi_0$. We then solve a series of problems, using $\phi_0 = 0$, and

$$\phi_1^{(i)} = (0,0,\ldots,1,0,..0) \qquad i = 1,\ldots.m$$

(i.e., $\phi_1^{(i)}$ has 1 in the i-th place and 0 elsewhere).

T is constructed in a simple manner from the solutions of these problems at $y = b$. The nonsingularity of T is precisely the criterion for the unique solvability of the entire problem.

Using this procedure, we calculated an example using $\phi(x, b) = \phi_2(x) = 1 + \sin x$ and $\phi(x,-a) = \phi_0(x) = 1 + \cos x$. The results are shown in Fig. 5. Again, the solid curves are the exact solutions from Sec. 2, and the circles are the computed results. The accuracy is again satisfactory.

Whether or not this procedure is more efficient than solving the ordinary differential equations is debatable, but this procedure is interesting in its own right, and permits a completely different way to look at the problem.

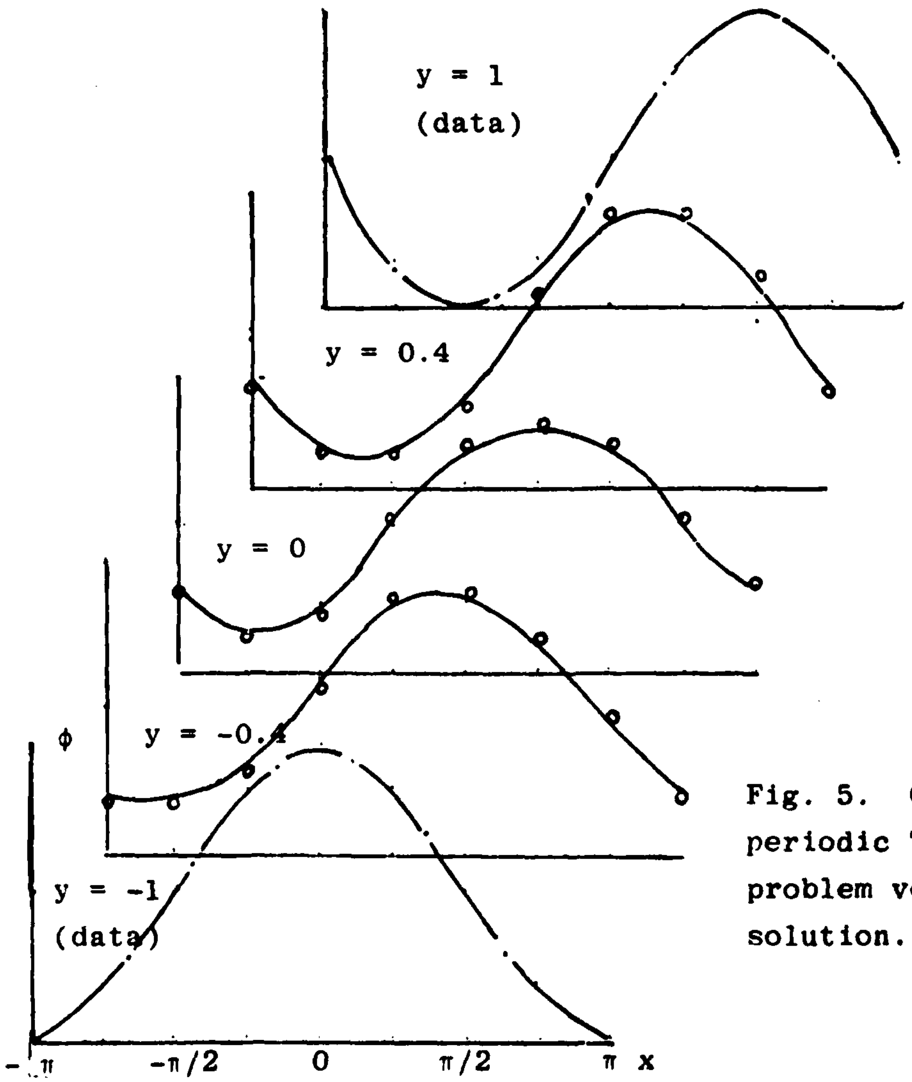

Fig. 5. Calculated periodic Tricomi problem versus exact solution.

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19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

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